Exam. Code : 103204
Subject Code : 9035

B.A./B.Sc. 4th Semester (Old Syllabus 2014)
MATHEMATICS
Paper—II
(Number Theory)

Time Allowed—Three Hours] [Maximum Marks—50

Note :— Attempt **FIVE** questions in all selecting at least **TWO** questions each from Sections A and B. All questions carry equal marks.

SECTION—A

I. (a) Let $a \in Z$. Show that a^2 leaves the remainder 0 or 1 when divided by 4 and hence show that 11111 is not perfect square.

(b) Show that $\frac{a(a^2+2)}{3}$ is an integer for all $a \geq 1$.

5,5

II. (a) Prove that $(a, m) = (b, m) = 1$ iff $(ab, m) = 1$.

(b) Prove that there are an infinite number of primes of the form $4n + 3$. 5,5

III. (a) Verify that $2^{2^5} + 1$ is divisible by 641.

(b) Prove that if $2^n - 1$ is a prime, then n is prime.

5,5

IV. (a) If $p \geq 5$ is a prime number, then show that $p^2 + 2$ is composite.

(b) Show that necessary and sufficient condition that a positive integer n can be divided by 3 is that the sum of its digits is divisible by 3.

5,5

V. (a) For any prime p, prove that

$$(a + b)^p \equiv a^p + b^p \pmod{p}.$$

(b) Find the general solution of $39x - 56y = 11$.

5,5

SECTION—B

VI. (a) For any prime number p, prove that

$$(p - 1)! \equiv -1 \pmod{p}.$$

(b) Solve the set of simultaneous congruencies $4x \equiv 3 \pmod{5}$, $5x \equiv 2 \pmod{6}$. 5,5

VII. (a) If $m > 2$, then prove that $\phi(m)$ is even.

(b) Find the least positive integer that gives remainder 1, 2, 3, when divided by 3, 4, 5 respectively.

5,5

VIII. (a) If $\tau(n)$ denotes the number of positive divisors of n, then show that

$$\prod_{d/n} d = n^{\tau(n)/2}, \text{ for an integer } n > 1.$$

(b) Find the highest power of 18 that is contained in 500 !. 5,5

IX. (a) For any positive integer $n \geq 1$, show that

$$\sum_{d/n} \phi(d) = n.$$

(b) Verify Mobius Inversion formula for n = 24. 5,5

X. (a) Prove that the function μ is multiplicative.

(b) Evaluate τ and σ for n = 3000. 5,5